

CHAPTER 11

THREE DIMENSIONAL GEOMETRY

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. What is the distance of point (a, b, c) from x -axis?
2. What is the angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$?
3. Write the equation of a line passing through $(2, -3, 5)$ and parallel to line $\frac{x-1}{3} = \frac{y-2}{4} = \frac{z+1}{-1}$.
4. Write the equation of a line through $(1, 2, 3)$ and perpendicular to $\vec{r} \cdot (\hat{i} - \hat{j} + 3\hat{k}) = 5$.
5. What is the value of λ for which the lines $\frac{x-1}{2} = \frac{y-3}{5} = \frac{z-1}{\lambda}$ and $\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z}{2}$ are perpendicular to each other.
6. If a line makes angle α , β , and γ with co-ordinate axes, then what is the value of $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$?
7. Write line $\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{j} - \hat{k})$ into Cartesian form.
8. If the direction ratios of a line are 1, -2, 2 then what are the direction cosines of the line?
9. Find the angle between the planes $2x - 3y + 6z = 9$ and xy - plane.
10. Write equation of a line passing through $(0, 1, 2)$ and equally inclined to co-ordinate axes.
11. What is the perpendicular distance of plane $2x - y + 3z = 10$ from origin?
12. What is the y -intercept of the plane $x - 5y + 7z = 10$?
13. What is the distance between the planes $2x + 2y - z + 2 = 0$ and $4x + 4y - 2z + 5 = 0$.
14. What is the equation of the plane which cuts off equal intercepts of unit length on the coordinate axes.
15. Are the planes $x + y - 2z + 4 = 0$ and $3x + 3y - 6z + 5 = 0$ intersecting?
16. What is the equation of the plane through the point $(1, 4, -2)$ and parallel to the plane $-2x + y - 3z = 7$?



17. Write the vector equation of the plane which is at a distance of 8 units from the origin and is normal to the vector $(2\hat{i} + \hat{j} + 2\hat{k})$.
18. What is equation of the plane if the foot of perpendicular from origin to this plane is (2, 3, 4)?
19. Find the angles between the planes $\vec{r} \cdot (\hat{i} - 2\hat{j} - 2\hat{k}) = 1$ and $\vec{r} \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) = 0$.
20. What is the angle between the line $\frac{x+1}{3} = \frac{2y-1}{4} = \frac{2-z}{-4}$ and the plane $2x + y - 2z + 4 = 0$?
21. If O is origin $OP = 3$ with direction ratios proportional to $-1, 2, -2$ then what are the coordinates of P ?
22. What is the distance between the line $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(\hat{i} + \hat{j} + 4\hat{k})$ from the plane $\vec{r} \cdot (-\hat{i} + 5\hat{j} - \hat{k}) + 5 = 0$.
23. Write the line $2x = 3y = 4z$ in vector form.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

24. The line $\frac{x-4}{1} = \frac{2y-4}{2} = \frac{k-z}{-2}$ lies exactly in the plane $2x - 4y + z = 7$. Find the value of k .
25. Find the equation of a plane containing the points (0, -1, -1), (-4, 4, 4) and (4, 5, 1). Also show that (3, 9, 4) lies on that plane.
26. Find the equation of the plane which is perpendicular to the plane $\vec{r} \cdot (5\hat{i} + 3\hat{j} + 6\hat{k}) + 8 = 0$ & which is containing the line of intersection of the planes $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) = 4$ and $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$.
27. If l_1, m_1, n_1 , and l_2, m_2, n_2 are direction cosines of two mutually perpendicular lines, show that the direction cosines of line perpendicular to both of them are

$$m_1n_2 - n_1m_2, n_1l_2 - l_1n_2, l_1m_2 - m_1l_2.$$



28. Find vector and Cartesian equation of a line passing through a point with position vectors $2\hat{i} + \hat{j} + \hat{k}$ and which is parallel to the line joining the points with position vectors $-\hat{i} + 4\hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 2\hat{k}$.
29. Find the equation of the plane passing through the point (3, 4, 2) and (7, 0, 6) and is perpendicular to the plane $2x - 5y = 15$.
30. Find equation of plane through line of intersection of planes $\vec{r} \cdot (2\hat{i} + 6\hat{j}) + 12 = 0$ and $\vec{r} \cdot (3\hat{i} - \hat{j} + 4\hat{k}) = 0$ which is at a unit distance from origin.
31. Find the image of the point (3, -2, 1) in the plane $3x - y + 4z = 2$.
32. Find the equation of a line passing through (2, 0, 5) and which is parallel to line $6x - 2 = 3y + 1 = 2z - 2$.
33. Find image (reflection) of the point (7, 4, -3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.
34. Find equations of a plane passing through the points (2, -1, 0) and (3, -4, 5) and parallel to the line $2x = 3y = 4z$.
35. Find distance of the point (-1, -5, -10) from the point of intersection of line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 5$.
36. Find equation of the plane passing through the points (2, 3, -4) and (1, -1, 3) and parallel to the x-axis.
37. Find the distance of the point (1, -2, 3) from the plane $x - y + z = 5$, measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$.
38. Find the equation of the plane passing through the intersection of two plane $3x - 4y + 5z = 10$, $2x + 2y - 3z = 4$ and parallel to the line $x = 2y = 3z$.
39. Find the distance between the planes $2x + 3y - 4z + 5 = 0$ and $\vec{r} \cdot (4\hat{i} + 6\hat{j} - 8\hat{k}) = 11$.
40. Find the equations of the planes parallel to the plane $x - 2y + 2z - 3 = 0$ whose perpendicular distance from the point (1, 2, 3) is 1 unit.



41. Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect each other. Find the point of intersection.
42. Find the shortest distance between the lines $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ and $\vec{r} = (2\hat{i} + 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 5\hat{k})$.
43. Find the distance of the point $(-2, 3, -4)$ from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$.
44. Find the equation of plane passing through the point $(-1, -1, 2)$ and perpendicular to each of the plane $\vec{r} \cdot (2\hat{i} + 3\hat{j} - 3\hat{k}) = 2$ and $\vec{r} \cdot (5\hat{i} - 4\hat{j} + \hat{k}) = 6$.
45. Find the equation of a plane passing through $(-1, 3, 2)$ and parallel to each of the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$.
46. Show that the plane $\vec{r} \cdot (\hat{i} - 3\hat{j} + 5\hat{k}) = 7$ contains the line $\vec{r} = (\hat{i} + 3\hat{j} + 3\hat{k}) + \lambda(3\hat{i} + \hat{j})$.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

47. Check the coplanarity of lines $\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda(-3\hat{i} + \hat{j} + 5\hat{k})$ and $\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \mu(-\hat{i} + 2\hat{j} + 5\hat{k})$.
If they are coplanar, find equation of the plane containing the lines.
48. Find shortest distance between the lines : $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.



49. Find shortest distance between the lines :

$$\vec{r} = (1 - \lambda)\hat{i} + (\lambda - 2)\hat{j} + (3 - 2\lambda)\hat{k}$$

$$\vec{r} = (\mu + 1)\hat{i} + (2\mu - 1)\hat{j} + (2\mu + 1)\hat{k}.$$

50. A variable plane is at a constant distance $3p$ from the origin and meets the coordinate axes in A, B and C . If the centroid of $\triangle ABC$ is (α, β, γ) , then show that $\alpha^{-2} + \beta^{-2} + \gamma^{-2} = p^{-2}$.

51. A vector \vec{n} of magnitude 8 units is inclined to x -axis at 45° , y axis at 60° and an acute angle with z -axis. If a plane passes through a point $(\sqrt{2}, -1, 1)$ and is normal to \vec{n} , find its equation in vector form.

52. Find the foot of perpendicular from the point $2\hat{i} - \hat{j} + 5\hat{k}$ on the line $\vec{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$. Also find the length of the perpendicular.

53. A line makes angles $\alpha, \beta, \lambda, \delta$ with the four diagonals of a cube. Prove that

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = \frac{4}{3}.$$

54. Find the equation of the plane passing through the intersection of planes $2x + 3y - z = -1$ and $x + y - 2z + 3 = 0$ and perpendicular to the plane $3x - y - 2z = 4$. Also find the inclination of this plane with xy -plane.

ANSWERS

1. $\sqrt{b^2 + c^2}$

2. 90°

3. $\frac{x-2}{3} = \frac{y+3}{4} = \frac{z-5}{-1}.$

4. $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + 3\hat{k})$

5. $\lambda = 2$

6. 2

7. $\frac{x-1}{0} = \frac{y+1}{2} = \frac{z}{-1}.$

8. $\pm \frac{1}{\sqrt{3}}, \mp \frac{2}{\sqrt{3}}, \pm \frac{2}{\sqrt{3}}$



9. $\cos^{-1} (6/7)$.
10. $\frac{x}{a} = \frac{y-1}{a} = \frac{z-2}{a}, a \in R - \{0\}$
11. $\frac{10}{\sqrt{14}}$
12. -2
13. $\frac{1}{6}$
14. $x + y + z = 1$
15. No
16. $-2x + y - 3z = 8$
17. $\vec{r} \cdot (2\hat{i} + \hat{j} + 2\hat{k}) = 24$
18. $2x + 3y + 4z = 29$
19. $\cos^{-1} \left(\frac{11}{21} \right)$
20. 0 (line is parallel to plane)
21. $(-1, 2, -2)$
22. $\frac{10}{3\sqrt{3}}$
23. $\vec{r} = \vec{o} + \lambda(6\hat{i} + 4\hat{j} + 3\hat{k})$
24. $k = 7$
25. $5x - 7y + 11z + 4 = 0$.
26. $\vec{r} \cdot (-51\hat{i} - 15\hat{j} + 50\hat{k}) = 173$
28. $\vec{r} = (2\hat{i} - \hat{j} + \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ and $\frac{x-2}{2} = \frac{y+1}{-2} = \frac{z-1}{1}$.
29. $x - 2y + 3z = 1$
30. $\vec{r} \cdot (8\hat{i} + 4\hat{j} + 8\hat{k}) + 12 = 0$ or $\vec{r} \cdot (-4\hat{i} + 8\hat{j} - 8\hat{k}) + 12 = 0$
31. $(0, -1, -3)$
32. $\frac{x-2}{1} = \frac{y}{2} = \frac{z-5}{3}$.
33. $\left(\frac{47}{7}, -\frac{18}{7}, \frac{43}{7} \right)$
34. $29x - 27y - 22z = 85$
35. 13
36. $7y + 4z = 5$



37. 1

38. $x - 20y + 27z = 14$

39. $\frac{21}{2\sqrt{29}}$ units.

40. $x - 2y + 2z = 0$ or $x - 2y + 2z = 6$

41. $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$

42. $\frac{1}{\sqrt{6}}$

43. $\frac{17}{2}$

44. $\vec{r} \cdot (9\hat{i} + 17\hat{j} + 23\hat{k}) = 20$

45. $2x - 7y + 4z + 15 = 0$

47. $x - 2y + z = 0$

48. $\frac{16}{7}$

49. $\frac{8}{\sqrt{29}}$

51. $\vec{r} \cdot (\sqrt{2}\hat{i} + \hat{j} + \hat{k}) = 2$

52. $\hat{i} + 2\hat{j} + 3\hat{k}, \sqrt{14}$

54. $7x + 13y + 4z = 9, \cos^{-1}\left(\frac{4}{\sqrt{234}}\right).$